TRR 154 | Mathematical Modelling, Simulation and Optimization Using the Example of Gas Networks

Existence of Equilibria in Energy Markets with Convex and Nonconvex Players

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Trends in Mathematical Modelling, Simulation and Optimisation: Theory and Applications
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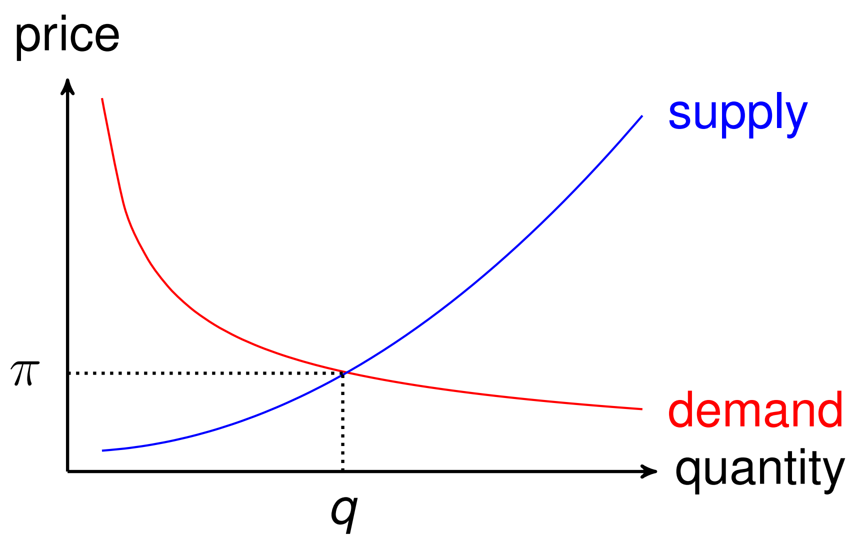
Motivation

Fundamental Welfare Theorems

Fundamental theorems of welfare economics (Arrow, Debreu, Walras)

Under suitably chosen assumptions (esp. convexity), competitive equilibria exist, are unique, and Pareto-optimal, i.e., there exists a 1-1 correspondence of market equilibria and welfare optima.



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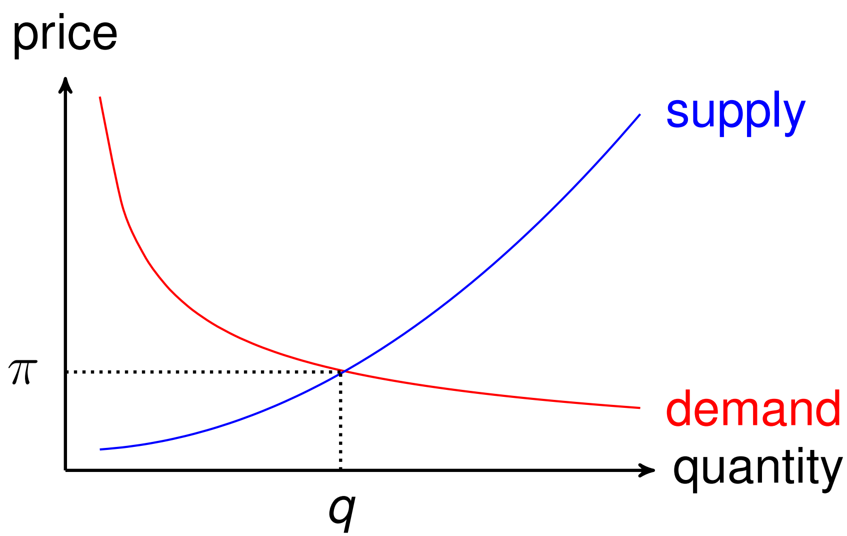

from https://gaslib.zib.de/data.html

Central Research Questions

Central research questions

Given an energy market game with **nonconvexities**.

1. Can we decide **existence of a competitive market equilibrium?**
2. How often exist equilibria for **respective applications?**

The Market Equilibrium Problem

Simultaneous Competitive Market Game (MEP)

All players $i \in I \ldots$

... are price-takers

... have perfect information

... solve the optimization problem

$$\min_{y_i} \quad f_i(y_i, \pi) := c_i(y_i) + \pi^T h_i(y_i) \quad \text{s.t.} \quad y_i \in Y_i$$

In addition, their best-responses satisfy the market clearing conditions

$$\sum_{i \in I} h_i(y_i) = 0$$

π	Price vector
f_i	Objective function of the player
y_i, Y_i	Decision variables, feasible set of the player

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The Corresponding Welfare Optimization Problem (WFP)

$$\min_{y} \quad \sum_{i \in I} c_i(y_i) \quad \text{s.t.} \quad y \in Y, \quad \sum_{i \in I} h_i(y_i) = 0$$

y	Decision variables of all players
Y	Cartesian product of individual feasible sets

Existence of Equilibria

Lagrangian Dual Problem of the (WFP)

$$\min_{y} \quad \sum_{i \in I} c_i(y_i) \quad \text{s.t.} \quad y \in Y, \quad \sum_{i \in I} h_i(y_i) = 0 \qquad \text{(WFP)}$$

$$\sup_{\pi} \; d(\pi) := \inf_{y \in Y} \; L(y, \pi) = \sum_{i \in I} \left(c_i(y_i) + \pi^T h_i(y_i) \right) \qquad \text{(LD-WFP)}$$

Theorem (See Part 1. of Theorem 2.3 in Harks (2020))

The pair (y^*, π^*) is a market equilibrium of (MEP) if and only if y^* and π^* are solutions of the welfare optimization problem (WFP) and the corresponding dual problem (WFP-LD), respectively, with zero duality gap.

Implications

Corollary

(a) *If* (y^*, π^*) *is an equilibrium of (MEP), then* y^* *is a global solution of (WFP).*

(b) *If* y^* *is a global solution of (WFP), for which there exists no* π *such that* (y^*, π) *is an equilibrium of (MEP), then (MEP) has no solution.*

(c) *If* (y^*, π^*) *and* $(\hat{y}, \hat{\pi})$ *are equilibria of (MEP), then so are* $(y^*, \hat{\pi})$ *and* $(\hat{y}, \pi^*)$*.*

Players with Unique Best-responses

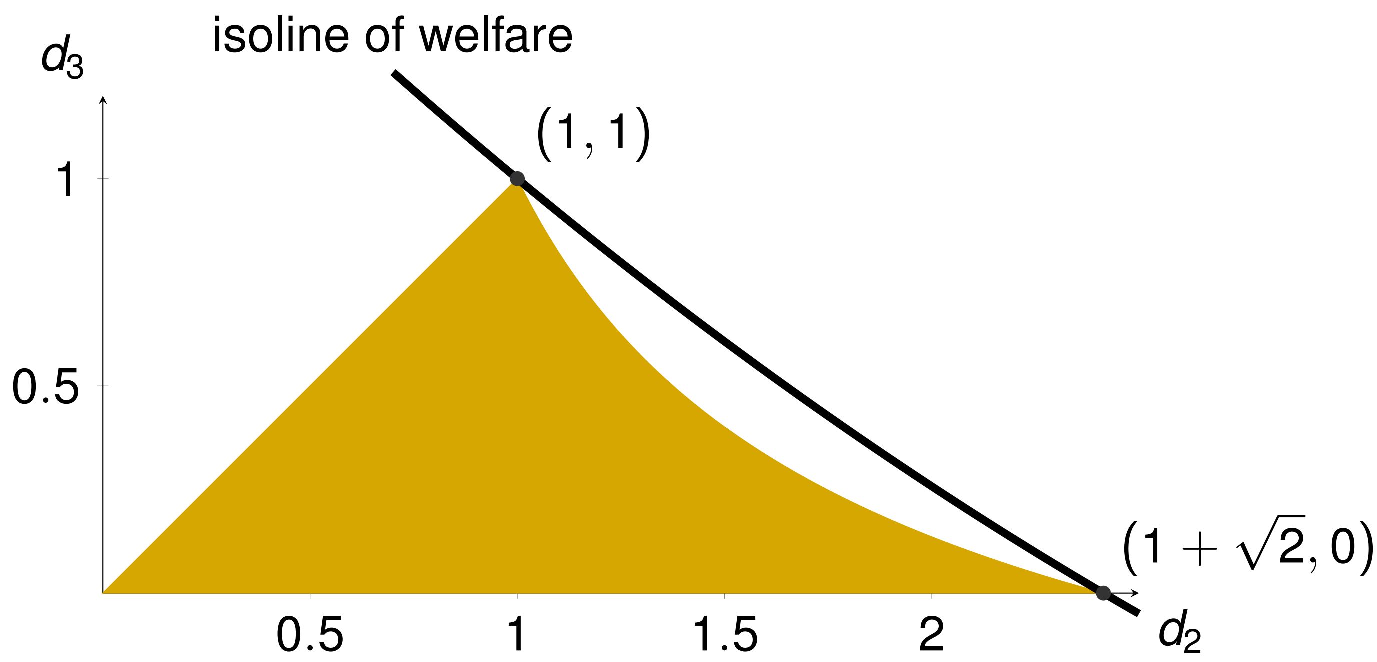
Corollary

Let $S \subseteq I$ be those players with unique best responses for all price vectors.

(a) *If (y^*, π^*) and $(\hat{y}, \hat{\pi})$ are equilibria of (MEP), then $y_S^* = \hat{y}_S$.*

(b) *If y^* and $\hat{y}$ are global solutions of (WFP) with $y_S^* \neq \hat{y}_S$, then (MEP) does not have a solution.*

Example: Non-existence due to Strictly Convex Players



from Grimm et al. (2019)

Equilibrium Price Candidate Set $\Pi(y^*) \subseteq \mathbb{R}^{n_\pi}$

$$(y^*, \pi^*) \text{ is a market equilibrium of (MEP)} \implies \pi^* \in \Pi(y^*)$$

Goal: Reduce candidate set to critical price vector $\hat{\pi}$!

$$\pi_k^- := \inf_{\pi \in \Pi(y^*)} \pi_k \quad \text{and} \quad \pi_k^+ := \sup_{\pi \in \Pi(y^*)} \pi_k \quad \text{for all} \quad k \in \{1, \dots, n_\pi\}$$

Identification of Critical Price Vector

Theorem

Let y^* be a solution of the (WFP) and let $\Pi(y^*) \neq \emptyset$ be given. Assume that for all $k \in \{1, \ldots, n_\pi\}$ at least one of the following properties is satisfied:

(a) $\pi_k^- = \pi_k^+$,

(b) $\pi_k^+ < \infty$ *and* $(h_i(y_i^*))_k \leq (h_i(y_i))_k$ *for all* $y_i \in Y_i$ *and all* $i \in I$,

(c) $\pi_k^- > -\infty$ *and* $(h_i(y_i^*))_k \geq (h_i(y_i))_k$ *for all* $y_i \in Y_i$ *and all* $i \in I$,

(d) $\pi_k^- = -\infty$, $\pi_k^+ = \infty$ *and* $(h_i(y_i^*))_k = (h_i(y_i))_k$ *for all* $y_i \in Y_i$ *and all* $i \in I$.

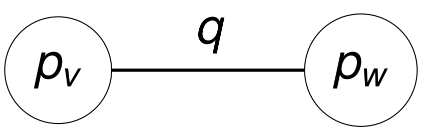
Then there exists an equilibrium of (MEP) if and only if $(y^*, \hat{\pi})$ *is an equilibrium, where* $\hat{\pi}$ *is defined as*

$$\hat{\pi}_k := \begin{cases} \pi_k^- = \pi_k^+, & \textit{if (a) applies,} \\ \pi_k^+, & \textit{if (b) applies,} \\ \pi_k^-, & \textit{if (c) applies,} \\ 0, & \textit{if (d) applies.} \end{cases}$$

Energy Market Applications and Numerical Results

Application “Nonlinear Stationary Gas Flow”

$$f_{\text{TSO}}(q) = \sum_{u \in V_- \cup V_+} \pi_u \left(\sum_{a \in \delta^-(u)} q_a - \sum_{a \in \delta^+(u)} q_a \right) - \sum_{a \in A} \alpha q_a q_a$$



$$p_v^2 - p_w^2 = \Lambda \left| q \right| q,$$
$$p_v^- \le p_v \le p_v^+, \quad p_w^- \le p_w \le p_w^+$$

Λ	Computed from gas and pipe parameters
q	Gas flow
p	Gas pressure

Test Instances for Application "Gas Physics"

- "GasLib - A Library of Gas Network Instances" from Schmidt et al. (2017)
- "Global Optimization for the Multilevel European Gas Market System with Nonlinear Flow Models on Trees" from Schewe et al. (2021)

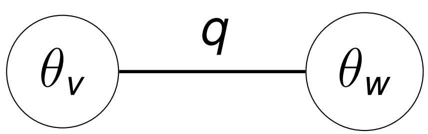
Instance	$\lvert V \rvert$	$\lvert V_- \rvert$	$\lvert V_+ \rvert$	$\lvert A \rvert$	$\#scenarios$	α
`GasLib-11 D5`	11	3	3	10	12	$\{0.01, 0.05, 0.1\}$
`GasLib-11`	11	3	3	11	20	$\{0.01, 0.05, 0.1\}$
`GasLib-24`	22	4	3	23	20	$\{0.01, 0.05, 0.1\}$
`GasLib-134`	123	41	3	122	20	$\{0.01, 0.05, 0.1\}$
`GasLib-134 TC`	134	45	3	133	20	$\{0.01, 0.05, 0.1\}$

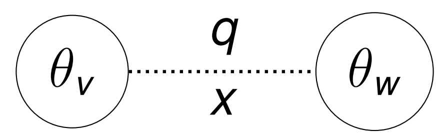
Numerical Results for Application “Gas Physics”

# instances	276
# instances solved within time limit	243
# instances with market equilibrium	243

Application "DC Line Switching"

$$f_{\text{TSO}}(q,x) = \sum_{u \in V_- \cup V_+} \pi_u \left(\sum_{a \in \delta^-(u)} q_a - \sum_{a \in \delta^+(u)} q_a \right) - \sum_{a \in A} \alpha q_a q_a - \sum_{a \in A_s} \beta (1 - x_a)$$





$$\theta_v - \theta_w = -\frac{1}{B} q$$

$$q^- \leq q \leq q^+$$

$$\left(\theta_v - \theta_w + \frac{1}{B} q \right) (1 - x) = 0, \quad qx = 0$$

$$q^- \leq q \leq q^+$$

B	Computed from pipe parameters
q, θ	Electricity flow, phase angle
x	Switching variable (0 on, 1 off)

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Test Instances for Application "DC Line Switching"

- "MATPOWER (Version 7.1)" from Zimmerman and Murillo-Sanchez (2020)
- 10% of all arcs are randomly selected as being switchable
- 42 scenarios in total
- Transport costs $\alpha \in \{0.1, 0.5, 1.0\}$
- Switching costs $\beta \in \{20, 50\}$

Instance	$\lvert V \rvert$	$\lvert V_- \rvert$	$\lvert V_+ \rvert$	$\lvert A \rvert$
Smallest	5	3	4	6
Average	2362	1246	405	3304
Biggest	13659	5043	4092	20467

Numerical Results for Application “DC Line Switching”

# instances	252
# instances solved within time limit	211
# instances with market equilibrium	121

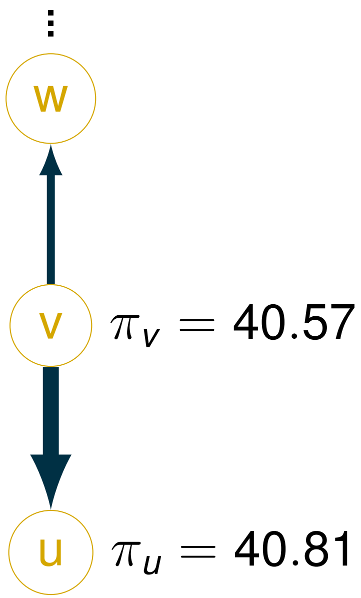
Observations

It becomes more likely that an equilibrium exists with . . .

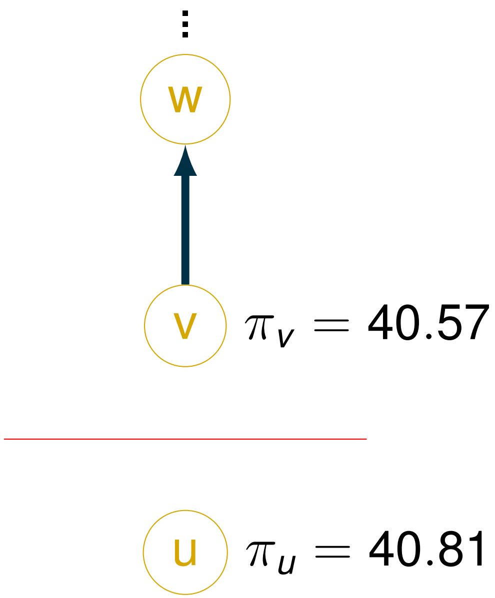
- . . . decreasing number of nodes
- . . . increasing transport costs
- . . . increasing switching costs

Example: Switched-off vs. Switched-on Consumers

Welfare solution:



Best-response of TSO:



Conclusion

Summary

Answers to the central research questions

Given an energy market game with **nonconvexities**.

1. Can we decide **existence of a competitive market equilibrium?**
   -
   -
   -
2. How often exist equilibria for **respective applications?**
   -
   -

Summary

Answers to the central research questions

Given an energy market game with **nonconvexities**.

1. Can we decide **existence of a competitive market equilibrium?**
   - Uniqueness / non-existence result for MEPs that include strictly convex players
   - Identification of critical price vector for specific types of MEPs occuring, e.g., in transportation networks
   - Algorithmic approach that decides existence of equilibrium and computes an equilibrium in case of existence
2. How often exist equilibria for **respective applications?**
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   - Algorithmic approach that decides existence of equilibrium and computes an equilibrium in case of existence
2. How often exist equilibria for **respective applications?**
   - Gas physics: Always
   - DC line switching: About 50% of cases

Thank you for your attention!

Questions + Answers