Recent methods for Optimal Power Flow with discrete variables

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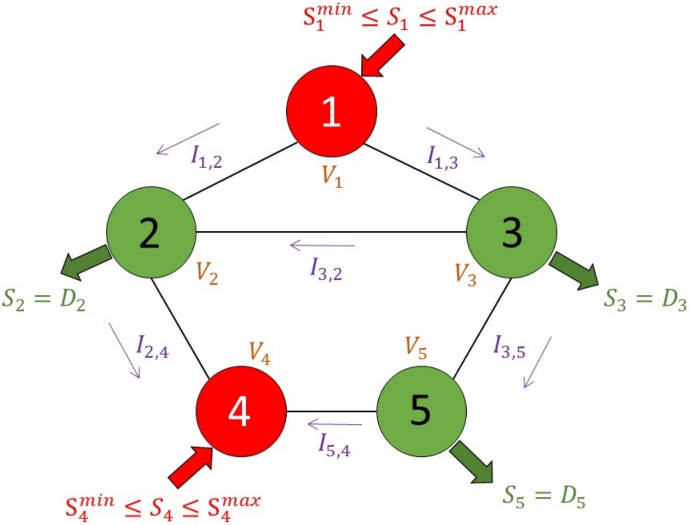
Example on a 5-nodes network



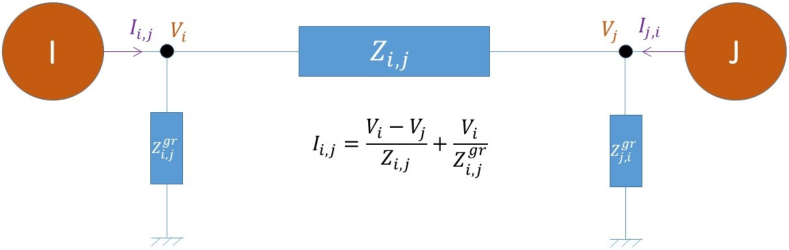
V, I, S are complex

The Alternative Current Optimal Power Flow problem

- Variables : voltages V, currents I and powers S.
- Physical laws :
  - Dictate connections between V, I and S.
- Power constraints :
  - Equalities on demanded power,
  - Bounds on generated power.
- Security constraints :
  - Bounds on voltage magnitude.
- Goal : minimize a cost, linearly linked to active generated power.

Complex variables model

Current is expressed from voltage :



- Power is expressed from voltage :

$$S_{i,j} = \overline{I_{i,j}} V_i = \left(\frac{1}{\overline{Z_{i,j}}} + \frac{1}{\overline{Z^{gr}_{i,j}}} \right) \overline{V_i} V_i - \frac{1}{\overline{Z_{i,j}}} \overline{V_j} V_i$$
$$S_{j,i} = \overline{I_{j,i}} V_j = \left(\frac{1}{\overline{Z_{i,j}}} + \frac{1}{\overline{Z^{gr}_{j,i}}} \right) \overline{V_j} V_j - \frac{1}{\overline{Z_{i,j}}} \overline{V_i} V_j$$

- Solving AC-OPF consists in determining a state of the network (i.e. the n complex voltage variables V).

QCQP model and properties

Complex model :

$$(OPF_{\mathbb{C}}) \begin{cases} \min & \overline{V}^T \tilde{A}_0 V \\ \text{s.t.} & \overline{V}^T \tilde{A}_k V \quad \leq \quad a_k, \quad k = 1, .., 6n \\ & V \in \mathbb{C}^n. \end{cases}$$

Real model :

$$(OPF) \begin{cases} \min & x^T A_0 x \\ \text{s.t.} & x^T A_k x \quad \leq \quad a_k, \quad k = 1, .., 6n \\ & x \in \mathbb{R}^{2n}. \end{cases}$$

- Some contraints are written as follows : $V_{min}^2 \leq x_i^2 + x_j^2$, and thus are non-convex,
- Problem AC-OPF is NP-hard [Bienstock and Verma 2015],
- Interior points methods find feasible solutions leading to sharp upper bounds,
- Conic relaxations give sharp lower bounds on the problem, especially semi-definite ones [Lavaei and Low, 2012].

What is AC OPF used for ?

- In many countries generation and transmission are not operated by the same company
- North America : economic dispatch. For only a part of generation
- Europe : no economic dispatch. Generation is mostly scheduled without transmission constraints because grid is strong and was built to serve load and generation.
- Europe : a lot of interconnections and exchanges within Europe. Market operation between countries are using power flow approximations (eg Flow Based Market Coupling)
- RTE in France : AC OPF is used for simulation purpose (long term optimal system, or short term voltage values for detailed studies)

US challenge

- Grid Optimization Competition
- https ://gocompetition.energy.gov/
- Up to $2.3 million in prizes for better power grid optimization !
- Prolem formulation is an issue

Interior point methods

- Works well
- Usually with polar representation of complex variables (sine and cosine)
- With carefull scaling and initial point choice
- Restart works well
- Fast solutions
- Excellent solutions
- RTE is using commercial software Knitro

SDP relaxations

- 2008 - 2012 SDP relaxation : problem is quadratic (non convex), replace $x_i x_j$ with w_{ij}. Also named rank relaxation.
- Excellent lower bounds
- Other conic relaxations, faster
- Improvements of bounds with cuts and bound tightening
- Problem 1 : what to do next ?
- Problem 2 : 1% optimality is not enough (transmission losses are 2%)

Global Optimality

- Lasserre hierarchy for small problems
- Partial Lasserre hierarchy for medium size problems
- JB Lasserre and V Magron are working for larger sizes
- Branch-and-Bound with SDP or conic relaxations, SDP or conic problem to solve at each node
- Branch-and-Bound using SDP optimization to compute an optimal quadratic convex reformulation (Hadrien Godard), quadratic QP at each node

Global optimality for continuous AC OPF is still open

Branch-and-bound iterations of the QP reformulation by Hadrien Godard

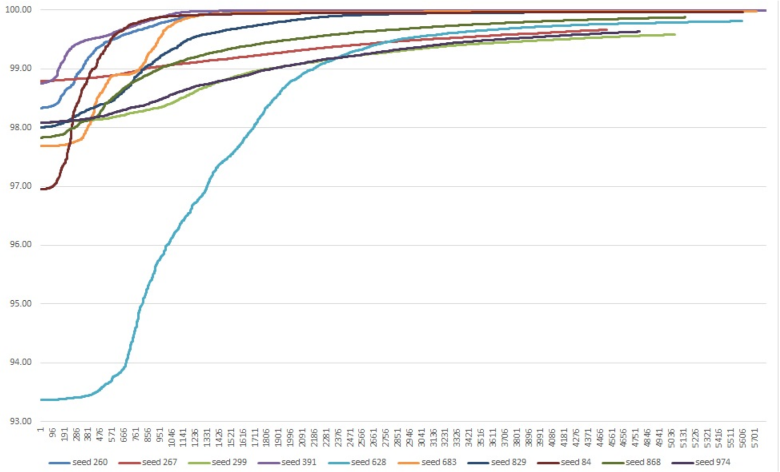


FIGURE – Gap closure during branch-and-bound iterations

Discrete decision variables

AC OPF is an academic very usefull problem to design, test and compare new algorithms and methods, but reality is more complex

- Topology choice : open or close a line, node splitting
- Phase Shifter Transformers (PST) taps choices
- Generating units : on/off status (Pmin > 0)
- Shunts connexion (capacitors, selfs), together with reactive power dispatch
- Taps of transformers : voltage transformer ratios have discrete values

- Works quite well with MPEC constraints
- x complements y : $x \geq 0$, $y \geq 0$, $xy = 0$
- Scaling and initial point are important
- More for finding feasible solutions than optimal solutions

Discrete modelling variables

Discrete variables are also used for multi state modelling

- N state : all lines are operating
- N-1 states : 1 line outage
- Modelling : if overload in N-1 state, then curative actions allowed
- Modelling : if N-1 state, then either voltage stays at N state value, or reactive power of generating unit is at maximum value (US challenge)

Discrete variables are also used for wost case modelling

Global optimality with discrete variables

- Lassere hierachy may always be used
- SDP and conic relaxation may be defined for every specific model using discrete variables

Conclusion

- Continuous AC OPF is quadratic non convex problem
- Interior point methods works well
- SDP, conic and Lasserre hierachy give lower bounds
- Global optimality efficient methods not yet available
- Discrete variables are waiting behind for both lower bounds and global optimality
- Real life also have time dimension

Global optimality quest

- Global Optimization : ϵ_1 for feasibility, ϵ_2 for (global) optimality, find solution for all instances (whatever long it takes)
- Not so important from economic point of view.
- Important when embedded in larger problems, eg as sub-problem of a much complex power system optimization (or simulation) problem

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