Subproject A05

Decomposition methods for mixed-integer optimal control

TRR 154
Mathematical modeling, simulation, and optimization using the example of gas networks

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A: Summary

The objective of this project is the development of mathematical algorithms to find an optimal control for mixed-integer problems on transport networks with the help of decomposition methods. The optimization problems are planned to be decomposed with respect to variables but also with respect to subsystems. On the upper level, there are integer decisions, while on the lower level the focus is on continuous variables. Additionally we want the subproblem to provide disjunctions for the masterproblem, because such an approach enables the algorithm to find global optima for non-convex problems as well. So the focus of the subproject A05 is on the mathematical analysis of structured MINLPs in the light of hierarchic models.

C: Decomposition Algorithm

- ▷ master problem: "easy" mixed-integer linear part (1a) and (1b)
- ▷ subproblem: nonlinear "black-box" functions (1c)

⇒ alternatingly solving relaxed MIPs (master problem) and separation problems (subproblem) for refining nonlinearities

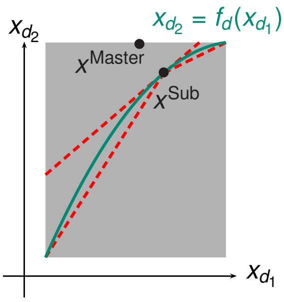
1st iteration

master problem:

- ▷ solve MIP with relaxed, linear set (gray) of (1c) ⇒ x^{Master}

subproblem:

- ▷ "black-box": yield closest, feasible solution to x^{Master} ⇒ x^{Sub}
- ▷ "black-box": create cuts (**red**) for next iteration



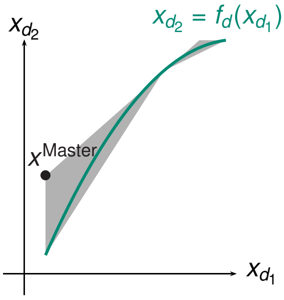
2nd iteration

master problem:

- ▷ solve MIP with relaxed, linear set (gray) of (1c) ⇒ x^{Master}

subproblem:

⋮



- ▷ stop criterion
  - ∘ MIP in master problem yields infeasibility ⇒ (1c) is infeasible
  - ∘ $\|x^{\text{Master}} - x^{\text{Sub}}\|_2^2 \le \varepsilon \Rightarrow x^{\text{Master}}$ is ε-feasible
- ▷ proof correctness (sketch)
  - ∘ assumption: functions f_d are strictly monotonic, stricly concave or convex, differentiable with bounded first derivative
  - ∘ x^{Master} is cut off for following iteration
  - ∘ sequence of solutions x^{Master} converges to ε-feasible solution

B: Problem Statement

$$\min_x \quad c^\top x \tag{1a}$$

$$\text{s.t.} \quad \underline{x} \le x \le \bar{x},\ x_{\mathcal{C}} \in \mathbb{R}^{|\mathcal{C}|},\ x_{\mathcal{I}} \in \mathbb{Z}^{|\mathcal{I}|},\ Ax \ge b, \tag{1b}$$

$$x_{d_2} = f_d(x_{d_1}) \quad \text{for all } d \in \mathcal{D}. \tag{1c}$$

- ▷ nonconvex MINLP with nonlinear "black-box" functions $f_d : \mathbb{R} \to \mathbb{R}$
- ▷ "black-box": no explicit knowledge required

D: Application in Stationary Gas Transport Optimization

- ▷ model for nomination validation
  - ∘ decide feasibility of supply & discharge flows
  - ∘ linear objective (1a): minimize compressor costs
  - ∘ variables and linear constraints (1b): binary variable for compressors/control valves, flow/density variables, linearized compressor model, control valve model, conservation of mass.
  - ∘ "black-box" functions f_d in (1c): connecting entry and exit density of pipes via Euler equations for compressible fluids:

$$\partial_x \left(p + \frac{q}{\rho} \right) = -\frac{1}{2}\frac{\lambda}{D}\frac{q|q|}{\rho}$$

λ : friction factor
D : pipe diameter
q : flow
ρ : density

- ▷ test instance: Greek natural gas network (`http://www.desfa.gr`)

topology:

3	entries ▲
45	exits ○
1	compressor ●
1	control valve ■
86	pipes

results:
(11/01/2011–02/17/2016)

	#	∅ It.	∅ Time (s)
opt.	1014	4.86	15.75
inf.	220	2.11	5.97

E: Work in Progress

- ▷ development for higher dimensional functions f_d in (1c)
- ▷ instationary aspects for gas transport optimization

F: Reference

M. Gugat, G. Leugering, A. Martin, M. Schmidt, M. Sirvent, D. Wintergerst - Towards Simulation Based Mixed-Integer Optimization with Differential Equations
`http://www.optimization-online.org/DB_HTML/2016/07/5542.html`

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