Subproject B07

Adaptive MIP Relaxations for MINLPs

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TRR 154

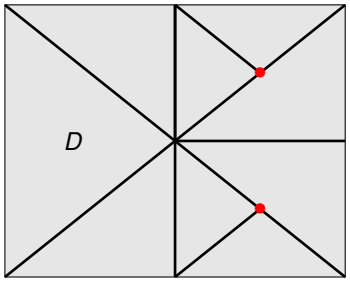
Mathematical modeling, simulation, and optimization using the example of gas networks

Problem Statement

Goal of the project is the analysis and solution of large-scale MINLPs, especially from the application of instationary gas network optimization, using adaptive MIP models. We approximate the nonlinearities with piecewise-linear functions to construct MIP relaxations of the underlying MINLP. In order to find a global optimum of the given MINLP an iterative algorithm is developed which solves a MIP relaxation and refines it subsequently.
In addition, theoretical results linking the complexity of the relaxations to structural properties of the nonlinear functions and the linearization error are of interest.

Refinement by Longest Edge Bisection

- refinement of a triangulation on domain D of a nonlinear function:



- preserving the structure of the triangulation
- discontinuity of approximation no problem since only a solution with given linearization error for MINLP problem is needed
- Burlacu, Geißler, Morsi, Martin, Schewe: *Using a Longest Edge Bisection to solve Mixed-Integer Nonlinear Programs by Mixed-Integer Linear Program relaxations.* In preperation.

Complexity

For a d-dimensional Lipschitz-continuous nonlinear function f with constant L_f, diameter $\operatorname{diam}(D_f)$ of box domain D_f and final linearization error ϵ_f at most

$$\mathcal{O}\left(2^{d^2}\frac{\epsilon_f}{L_f \operatorname{diam}(D_f)}\right)$$

refinement steps are performed. The number of binary variables is of size $\mathcal{O}\left(d^2\left|\ln\left(\frac{\epsilon_f}{L_f \operatorname{diam}(D_f)}\right)\right|\right)$.

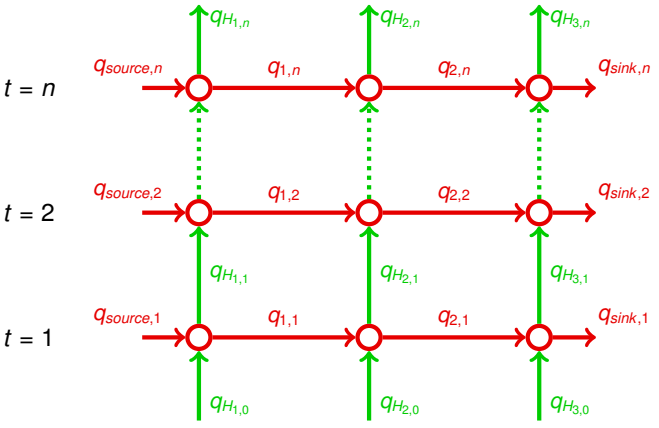
Implementation Strategies (WIP)

Exploiting the structure preserving property of the Longest Edge Bisection:

- solve a coarse MIP relaxation with a branch-and-bound algorithm
- perform a Longest Edge Bisection by constraint branching on nodes with incumbent solutions
- get finer solutions from incumbent solutions with warm starts

Instationary Gas Networks

We extend stationary gas networks via time-expanded graphs and discretization of time (model from subproject A07):



Benefits and challenges:

- low dimensional nonlinear functions (stationary gas networks)
- error analysis for piecewise-linear approximations of underlying nonlinear functions
- adaptive time discretizations depending on pipe properties

Challenges

- refinement strategies for MIP relaxations of MINLPs with focus on warm starts
- exploiting problem specific structures arising in instationary gas network optimization problems